# 5.6 Inequalities in Two Triangles and Indirect Proof

Theorem 5.13-Hinge Theorem--If 2 sides of one triangle are congruent to 2 sides of another triangle, and the included angle of the 1st triangle is greater than the included angle of the 2nd triangle, then the 3rd side of the 1st triangle is greater than the 3rd side of the 2nd triangle.

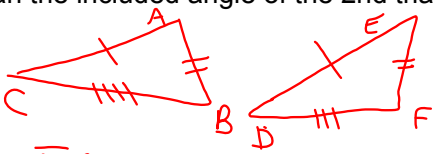

gsp demo

G: $\overline{AC} \cong \overline{DE}$ $\overline{AB} \cong \overline{EF}$

$m\angle A > m\angle E$

Concl: $CB > DF$

Theorem 5.14-Converse of the Hinge Theorem--If 2 sides of one triangle are congruent to 2 sides of another triangle, and the 3rd side of the 1st triangle is greater than the 3rd side of the 2nd triangle, then the included angle of the 1st triangle is greater than the included angle of the 2nd triangle.

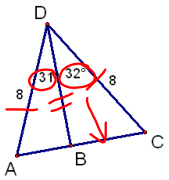



G: $\overline{AC} \cong \overline{DE}$ $\overline{AB} \cong \overline{EF}$

$CB > DF$

Concl: $m\angle A > m\angle E$

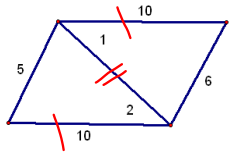
Compare the listed sides or angles.




BC > AB

32 31

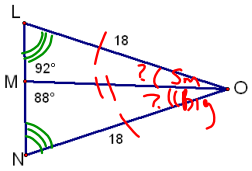
Compare the listed sides or angles.




m∠1 > m∠2

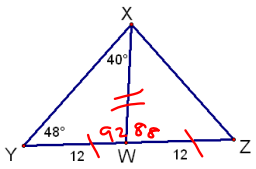
6 5

Compare the listed sides or angles.




LM < MN

Compare the listed sides or angles.

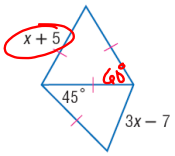



XZ < XY

88 92

Write an inequality to describe the possible values of $x$.

5.




$x+5>0$

$x>-5$

$3x-7>0$

$x>\frac{7}{3}$

$x+5>3x-7$

$12>2x$

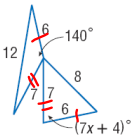
$6>x$

$x<6$

$\frac{7}{3}<x<6$

Write an inequality to describe the possible values of $x$.

6.




$7x+4 < 140$

$7x < 136$

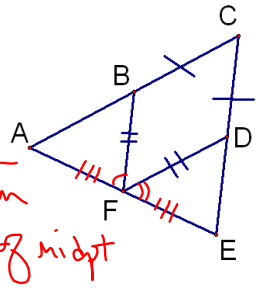
$x < \frac{136}{7}$

$7x+4 > 0$

$x > -\frac{4}{7}$

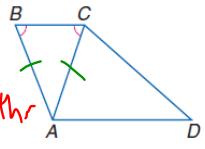
$-\frac{4}{7} < x < 19\frac{3}{7}$

Given: $\overline{BF} \cong \overline{DF}$; $\overline{BC} \cong \overline{CD}$;
F is the midpoint of $\overline{AE}$;
$m\angle DFE > m\angle AFB$
Prove: $CE > AC$




| S. | R. |
|---|---|
| 1) ~ | 1) Given |
| 2) $\overline{AF} \cong \overline{FE}$ | 2) def of midpt |
| 3) $DE > AB$ | 3) Hinge thm |
| 4) $AB+BC=AC$, $CD+DE=CE$ | 4) SAP |
| 5) $BC=CD$ | 5) def of $\cong$ |
| 6) $CD+DE > AB+BC$ | 6) Add. |
| 7) $CE > AC$ | 7) Subst. |

38. Given: $\angle B \cong \angle ACB$
Prove: $AD + AB > CD$




| S. | R |
|---|---|
| 1) ~ | 1) Given |
| 2) $AD+AC > CD$ | 2) $\triangle$ Ineq thm |
| 3) $\overline{AB} \cong \overline{AC}$ | 3) Conv. BAT |
| 4) $AB = AC$ | 4) def of $\cong$ |
| 5) $AD+AB > CD$ | 5) Subst |

Indirect Proof

1. Assume conclusion is false
2. Reason until you contradict the given
3. State assumption is false

Also called proof by contradiction.

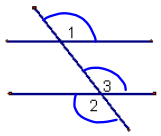
Example 1
Given: Mary received an A on the test.

Prove: Her grade was $\geq$90%.

Assume Mary earned an 89.
Then she would have received a B, which contradicts our given.
Our assumption is false, Mary's grade was $\geq$ 90%.

Example 2
Given: $\angle 1 \cong \angle 2$

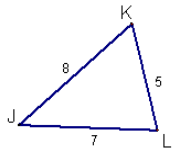
Prove: $\angle 1 \cong \angle 3$




① Assume $\angle 1 \cong \angle 3$.
② then $\angle 3 \cong \angle 2$ (vert. $\angle$s $\cong$)
then $\angle 1 \cong \angle 2$ (transitive)
* Contradicts our given
③ Our assumption is false
$\therefore \angle 1 \ncong \angle 3$
Therefore

Example 3
Given: picture

Prove: m$\angle$K < m$\angle$L

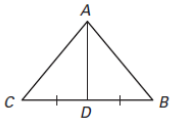



① Assume m$\angle$K > m$\angle$L
② then JL > JK (thm 5.11)
*Contradicts given
Assume m$\angle$K = m$\angle$L
then JL = JK (converse BAT)
*Contradicts given

**Indirect Proof** Arrange statements A–F in order to write an indirect proof of Case 1.

**GIVEN:** $\overline{AD}$ is a median of $\triangle ABC$.
$\angle ADB \cong \angle ADC$

**PROVE:** $AB = AC$




**Case 1:**

**A.** Then $m\angle ADB < m\angle ADC$ by the converse of the Hinge Theorem.
**B.** Then $\overline{BD} \cong \overline{CD}$ by the definition of midpoint. Also, $\overline{AD} \cong \overline{AD}$ by the reflexive property.
**C.** This contradiction shows that the temporary assumption that $AB < AC$ is false.
**D.** But this contradicts the given statement that $\angle ADB \cong \angle ADC$.
**E.** Because $\overline{AD}$ is a median of $\triangle ABC$, $D$ is the midpoint of $\overline{BC}$.
**F.** Temporarily assume that $AB < AC$.

Case 2:

Example 4

Given: $\frac{1}{2y + 4} = 20$

Prove: $y \neq -2$

HW p338-340
#s 1-9, 11-13, 16-18, 24

## Attachments

Hinge_thm.gsp